UNIT - 4

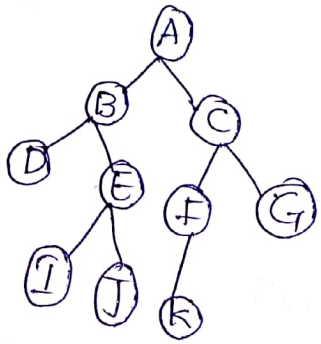
TREES

* A tree is a non-linear data structure in which the elements are organised in the form of hierarchical structure.

* A tree is a collection of elements (or) nodes and some associated operations.

* Hence the tree can also be called as an abstract data type.

Eg:-



Terminology of tree:-

1. Root:- In the tree, the topest node can be called as Root (or) Root node.

* Every tree must contain only one root node.

2. Edge:- In the tree, a connecting link between any two nodes is known as an edge, If the tree contains 'n' no.of nodes then the tree contains n-1 no.of edges,

3. Parent:- In the tree, If a node contains child (or) children then such node can be called as a parent (or) parent node.

4. child:- In the tree, If a node is having parent node then such node can be called as a child

5. Leaf :- In the tree, If a node does not have child (or) children then such node can be called as leaf (or) leaf node.

* leaf nodes can also be called as external nodes (or) terminal nodes.

6. Internal nodes :- In the tree, except the leaf nodes the remaining nodes can be called as Internal nodes

* the Internal nodes can also be called as non-terminal nodes.

7. Degree :- In the tree, the total no. of childs of a particular node is said to be degree of that particular node.

* Among all the nodes the node which is having highest degree can be treated as the degree of the entire tree.

8. Height :- In the tree, the total no. of edges from any leaf node to the specific node is said to be the height of that particular node

* the height of the entire tree is the height of the root node.

* The height of the leaf nodes is 'zero'

9. Depth :- In the tree, the total no. of edges from the root node to any particular node is said to be depth of that node.

* the heighest depth of a node can be treated as the entire depth of the tree.

* The depth of the root node is zero.

10. Level:- In the tree, usually the root node is represented at level 'zero' and its children are represented at level 'one' and their children will be represented at level 'two'. and so...

11. Path:- The sequence of consecutive edges from one node to another node is known as a path

* The length of the path is equal to the total no. of nodes existed along that particular path.

12. Sub tree:- In the tree, A sub tree can be formed by using child (or) children of a particular node

13. Anchestors & Descendants:- In the tree, all the nodes from root node to specific node are said to be anchestors for that node and all the nodes from the specific node to any leaf node are said to be descendants for that node.

14. Siblings:- In the tree, the nodes which are having the same parent node are said to be sibling

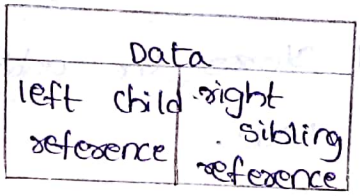
General Tree representation:-

Usually the general tress are represented using two ways they are 1. left child and 2) right sibling representation

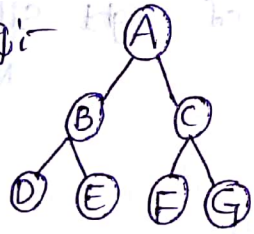
left child and right sibling representation:

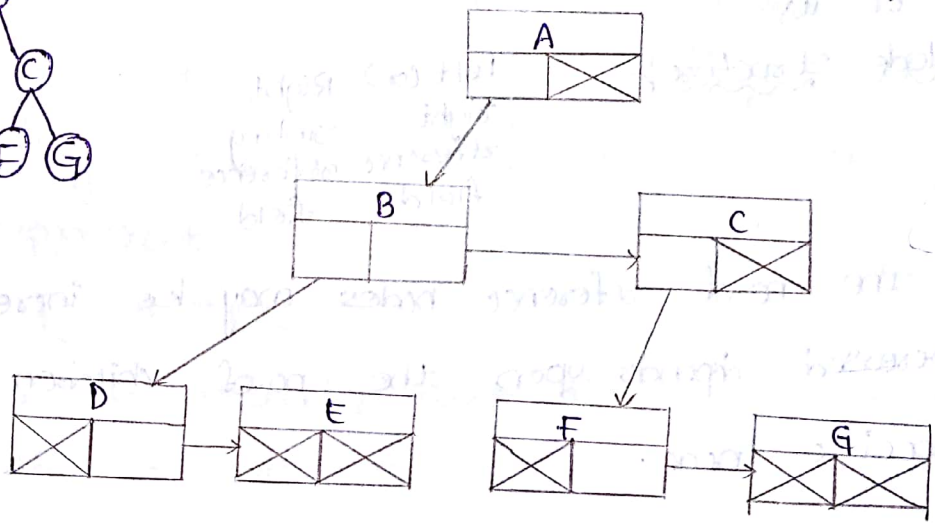
* In this representation, we use a special node which contains three fields such as data, left child reference and right sibling reference fields.
* The data field stores the actual element (or) value.
* The left child reference field stores the address of left child of the specific node
* The right sibling reference field stores the address of right sibling of the particular node.

Node structure:-



Eg:-





'X' indicates NULL.

List representation:-

In this representation we use two special node such as data node and reference node.

Data Node:- The data node contains 2 fields.

* The first field stores the actual element (or) value.
the second field stores the address of either left (or) right child.

Node structure:-

Data field	left (or) right child field

Reference Node:-

The reference node contains two fields

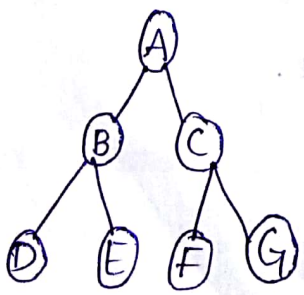
* The first field stores the address of either left (or) right child of a specific node.
* The second field stores the address of right sibling of its Node

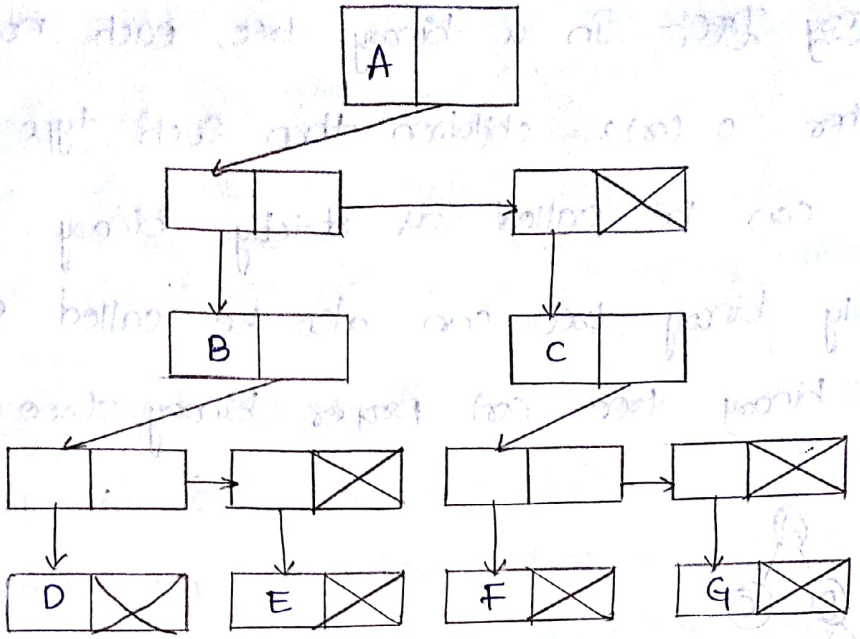
Node structure:-

left (or) right reference field	Right sibling reference field

* The no.of reference nodes may be increased (or) decreased depends upon the no.of children of a specific node.

Eg:-

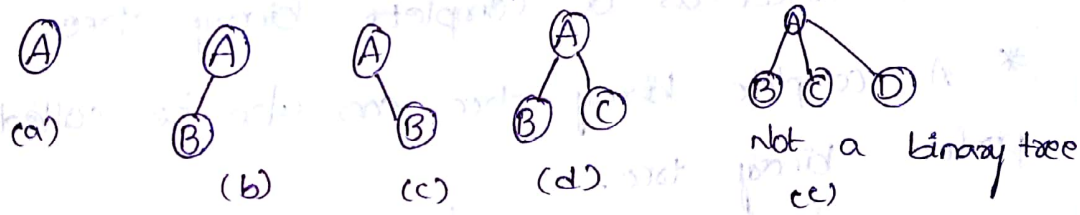




Binary Trees:-

* In a tree, if every node contains either '0' (or) '1' (or) '2' children but not more than '2' children then such type of binary tree can be called as a binary tree

Eg:-



* the binary trees are widely used to represent algebic expressions

Types of binary Trees:-

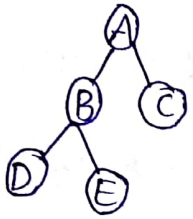
Based on the frequently usage the binary trees are classified into 5 types. they are.

1. strictly binary tree
2. complete binary tree
3. Extended binary tree
4. Binary Search tree
5. threaded binary tree.

Strictly binary Tree:- In a binary tree, each node contains either 0 (or) 2 children then such type of binary tree can be called as strictly binary tree.

* The strictly binary tree can also be called as either full binary tree (or) proper binary tree (or) 2-tree

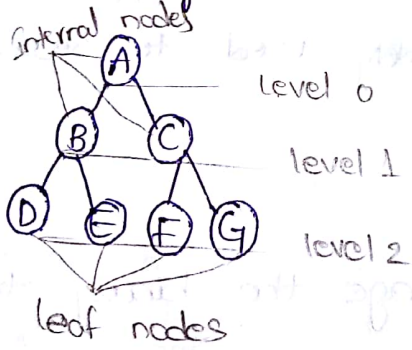
Eg:-



Complete Binary Tree:- In a binary tree, if every node contains exactly 2 children and all the leaf nodes are at same level then such type of binary tree can be called as a complete binary tree.

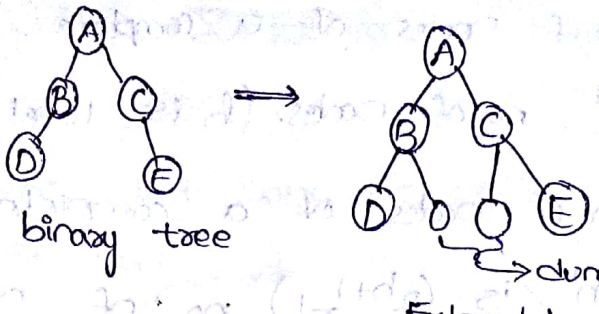
* A complete binary tree can also be called as a perfect binary tree.

Eg:-



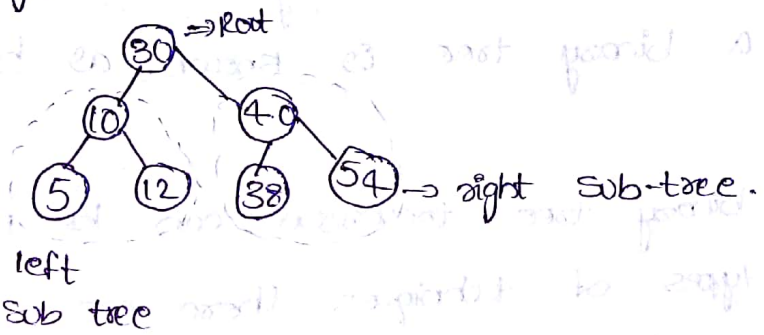
Extended binary tree:- If a binary tree is converted into strictly binary tree by adding some dummy nodes to the existing nodes then the converted strictly binary tree can be called as an extended binary tree.

Eg:-



* Binary Search Tree:- In a binary tree, the values in of all the nodes in the left sub-tree are less than the root node and the values of all the nodes in the right sub-tree are greater than (or) equal to the root node then such type of binary tree can be called as a binary search tree.

Eg:-



Threaded Binary Tree:- When a binary tree is implemented when in the form of linked list then we may get NULL values in some of the fields of the nodes if NULL values are replaced with some addresses of the nodes in that particular binary tree then such type of binary tree can be called as a threaded binary tree.

Binary Tree Properties:-

1. If the binary tree contains 'N' no.of nodes then it contains N-1 no.of edges.
2. In the binary tree, every node contains only one parent node except the root node.

3. The maximum no.of nodes of a complete binary tree at level 'l' is 2^l no.of nodes. (l is level number)

4. The maximum no.of nodes of a complete binary tree with height 'h' is $(2^{h+1}-1)$ no of nodes

5. The total no.of leaf nodes of a complete binary tree is $(n+1)/2$ no.of nodes (n = total no.of nodes)

6. The total no.of external nodes = total no.of internal nodes + 1.

Binary Tree Traversal:-

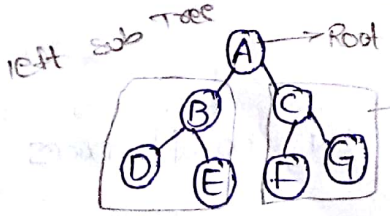
* The process of visiting and displaying every node in a binary tree is known as binary tree traversal

* the binary tree traversal can be performed by using 3 types of techniques there are

1) In-ordered Traversal
2) Pre-ordered Traversal
3) post-ordered Traversal.

In-ordered Traversal:- In this technique first the left sub-Tree will be traversed (or) visited and then root node and last the right sub Tree will be traversed. In the binary tree.

Eg:-



In-ordered :- ~~traversal~~ D-B-E-A-F-C-G

pre-ordered traversal:-

procedure steps:-

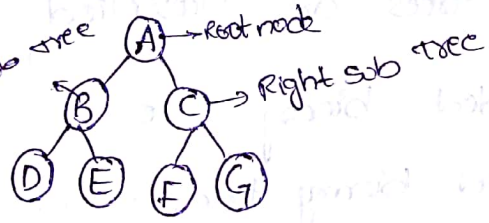
1. Traverse left sub-Tree
2. Traverse root node
3. traverse right sub-Tree.

pre-ordered Traversal:- In this technique first the root node is visited and then left sub-Tree and last the right sub-tree will be visited in the binary Tree.

procedure Steps:-

1. Transverse root node
2. Transverse left sub-Tree
3. Transverse right sub-Tree.

Eg:-



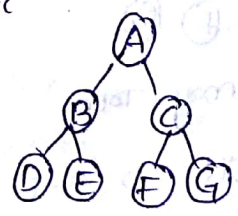
pre-order:- A - B - D - E - C - F - G

post-order Traversal:- In this technique first the left sub-Tree is will be traversed and then the right sub-Tree and last the root node will be traversed in the binary Tree.

procedure steps:-

1. Transverse left sub-Tree
2. Transverse right sub-Tree
3. Transverse root-node.

Eg:-



pre-order:-

D - E - B - F - G - C - A

Threaded Binary Tree:- If a binary tree is implemented in the form of double linked list then we may get NULL values in some fields of the nodes. If the NULL values are replaced with the addresses of some appropriate node in that binary tree then such binary tree can be called as a threaded binary tree.

The NULL value in the left link field of a node is replaced with predecessor node address of that particular node in the inorder form similarly the NULL value in the right link field of a node is replaced with sucessor node address of that particular node in the inorder form.

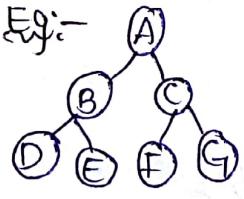
The threaded binary trees are classified into two types i) one-way threaded binary tree

ii) two-way threaded binary tree.

One-way Threaded Binary Tree:-

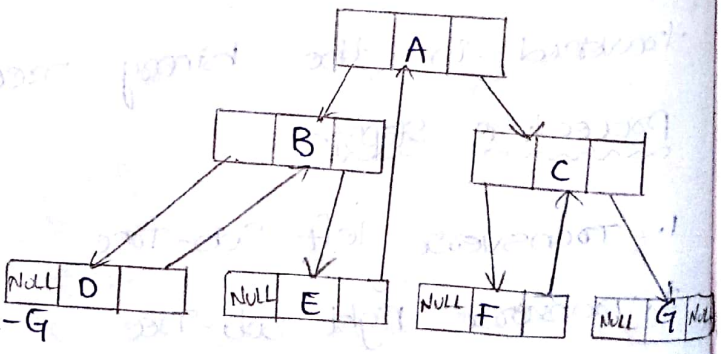
In this only the NULL value stored in the right link field of a node is replaced with appropriate node address in the binary tree

Eg:-



Binary Tree

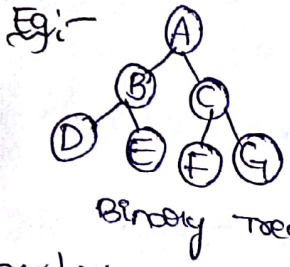
In-order:- D-B-E-A-F-C-G



One-way threaded binary tree

* TWO-WAY threaded Binary Tree:-

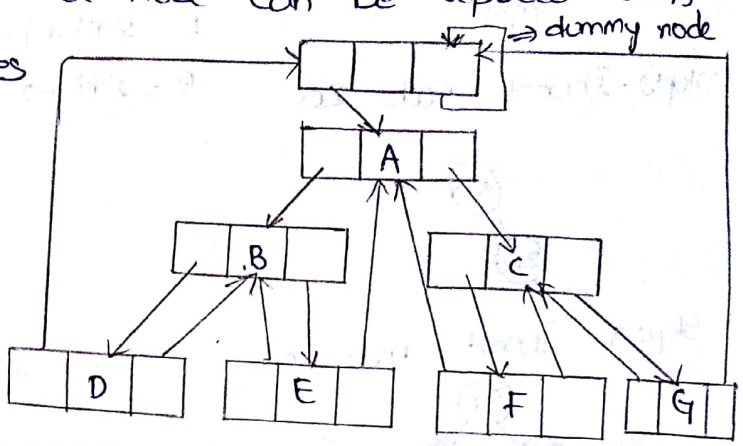
In this the NULL values stored in the left link and right link fields of a node can be replaced with appropriate nodes addresses



Binary Tree

Inorder:-

D-B-E-A-F-C-G



Two-way threaded binary tree

Heap (or) Binary Heap:-

A heap is a tree based data structure in which the elements are arranged in the form of hierarchical structure

The heap follows two basic properties they are.

i) shape (or) structure property

ii) heap property

Shape property:- this property states that the heap must be either complete (or) almost complete binary tree. In the complete binary tree the elements are organised from top to bottom and left to right in each level.

Heap property:- this property states that all the parent nodes in the heap must be either greater than (or) less than to their children.

* Usually the heaps are classified into two types they are i) max heap ii) Min heap.

Max heap:- In a heap if all the parent nodes are greater than their children then such heap can be called as Max heap.

Insertion

Eg:- construct a Max heap by the following elements.

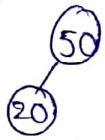
50, 20, 10, 40, 100.

Step:1 - Insert node 50

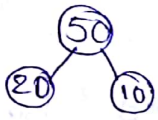
$L = 2*i + 1 = 1$

$R = 2*i + 2$

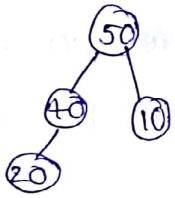
Step:2 - Insert node 20



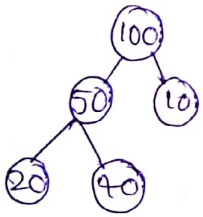
Step: 3 Insert node 10



Step:4 - Insert node 40

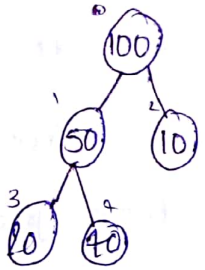


Step: 5 Insert node 100

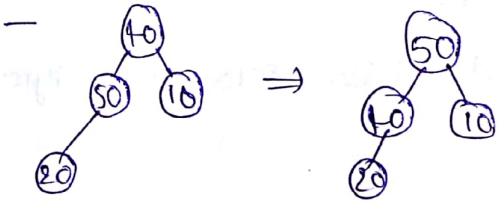


Deletion:-

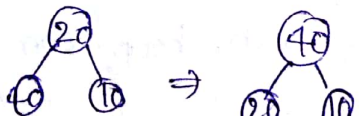
Eg:- Delete the element from the above Max heap.



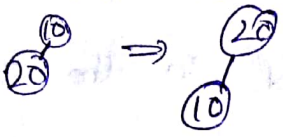
Step (1):-



(2) delete node 50



(3) delete node 40



4) delete node 20.

5) delete node 10

Min heap:- In a heap all the parent nodes are less than their children then such heap can be called as Min heap.

Priority Queue:- A priority queue is a linear data structure

Priority Queue:- A priority queue is a linear data structure in which the elements are stored in the sequencial order.

* A priority queue is a special queue in which the elements are stored in the order as they arrive along with their priorities and the elements will be deleted depends upon either max priority (or) Min priority.
* Usually the priority queues can be used in cpu shedduling
* the priority queue is classified into two types they are
1) Max priority queue
2) Min priority queue.

Max priority queue:- In this the elements are stored in the order as they arrive along with priorities and the elements are deleted depends upon highest priority.

Eg:- Insert the following elements into the max priority queue elements are 40, 20, 30, 10 and priorities 5, 6, 4, 2 respectively.

PQ[4]

	0	1	2	3
Data	40	20	30	10
priority	5	6	4	2

MAX priority queue

Max priority queue deleted

40	30	10
5	4	2

Min priority queue:- In this the elements are stored in the order as they arrive along with priorities and the elements are deleted depends upon lowest priority.

Eg:- Insert the following elements into the Min priority queue.

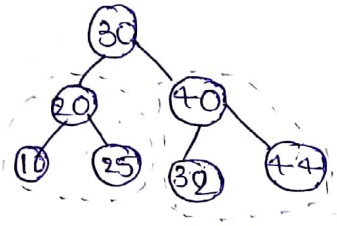
Min priority elements are. 40, 20, 30, 10 and priorities 5, 6, 4, 2 respectively.

PQ[4]

Data	40	20	30	10
priority	5	6	4	2

Binary search Trees:- In a binary tree if the values of all the nodes in the left sub-tree are lessthan the root node and the values of all the nodes in the right sub tree are greaterthan (or) equal to the root node

Eg:-



In the Binary search tree we perform 5 basic operation they are

1. create()
2. Insert()
3. Delete()
4. Search()
5. Display()

Time complexity:- the binary search takes O(log n) time complexity while performing either search (or) insert (or) delete operation

create:- this operation creates an initial node in the binary search tree.

Eg:- (30)

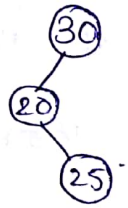
Insert:- this operation inserts a new node to the binary search tree

Eg:- Insert the following elements into the binary search tree.

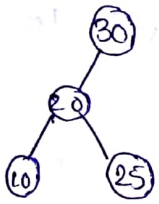
20, 25, 10, 40, 39, 45

1. Insert node 20

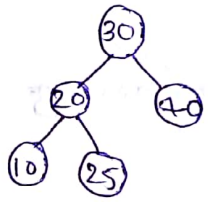
2. Insert node 25



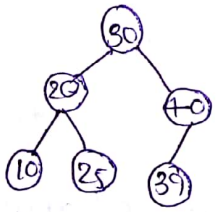
3. Insert node 10



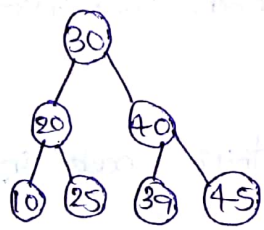
4. Insert node 40.



5. Insert node 39



6. Insert node 45



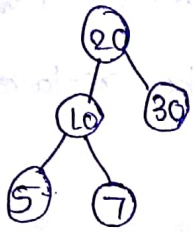
Delete :-

In the binary search tree the deletion can be performed in 3 possible cases they are.

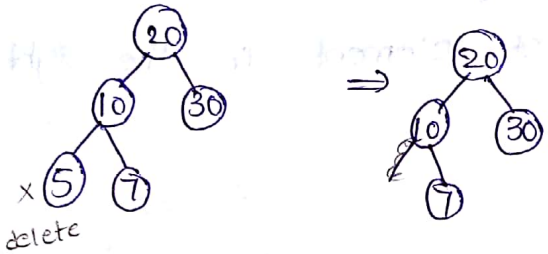
* case (1) Deleting a leaf node
case (2) Deleting a node having only one child
case (3) Deleting a node [illegible]

case(1): In this case first we perform the search operation to find out the specific leaf node. and then the leaf node can be deleted by removing its connection from its parent node.

Eg:-



Delete node 5

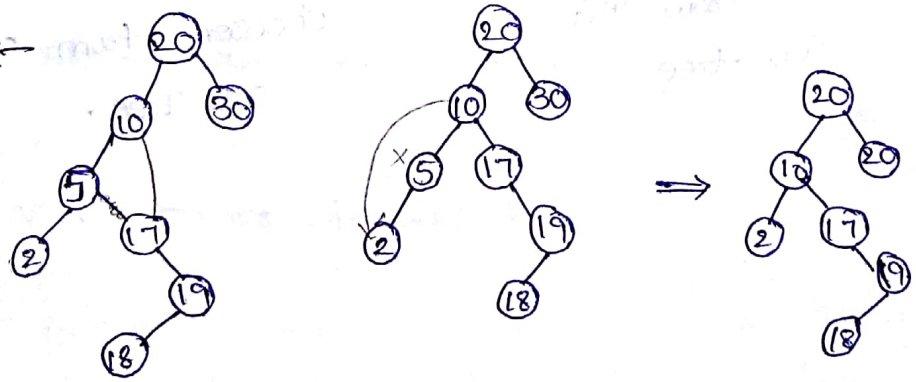


case(2): Deleting a node having only one child.

* In this case first we perform search operation to locate specific element for the deletion and then the connection will be established between child node and parent node of the specific node.

* In other words a node can be deleted having only one child by establishing the connection between its child node and parent node

Eg:-



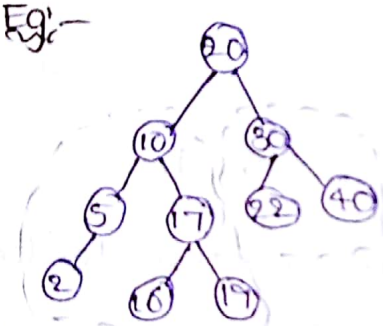
Delete node 5

Case(3): Deleting a node having 2 children:-

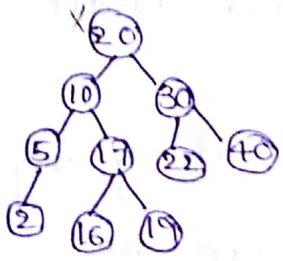
* In this case first we perform the search operation to locate the specific element for the deletion and then the highest element from the left sub-tree (or) lowest element from the right sub-tree can be replaced to the specific node for its deletion.

* In other words the node which is having 2 children can be deleted by replacing it with highest element in the left sub-tree (or) lowest element in the right sub-Tree of that specific node.

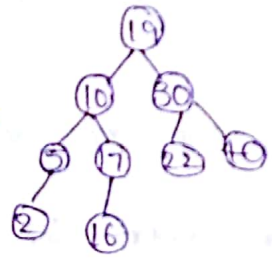
Eg:-



Delete node 20

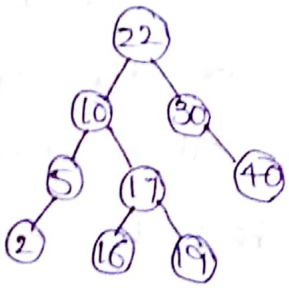


→



highest elements choosen from left sub-tree

(or)



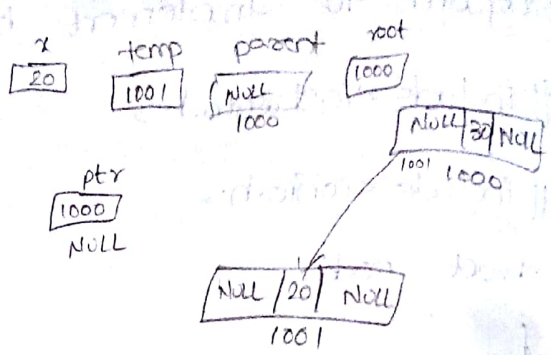
less element is choosen from right sub-Tree.

program to Implement binary search tree

```
#include <iostream.h>
#include <conio.h>
struct node
{
 int data;
 node *left;
 node *right;
}*root;
class bstree.
{
 public:
 void create( );
 void insert( );
 void in_order(node *);
 void pre_order(node *);
 void post_order(node *);
 void search(int);
 };
 void bstree::create( )
 {
 root = new node;
 cout<<"\n enter the data for the root node:";
 cin >> root ->data ;
 root -> left =root ->right = NULL ;
 }
 void bstree::insert( )
 {
  int x;
  node *temp, *parent ;
  cout<<"\n enter the element to be inserted:";
```

global structure.

```
cin>>x;
temp =new node;
temp->data = x;
temp -> left = NULL;
temp -> right = NULL;
parent = NULL;
if (root == NULL)
 {
 root = temp;
 }
else
 {
 node  *ptr;
 ptr = root;
 while(ptr != NULL)
 {
  parent =ptr;
  if (x > ptr -> data)
  parent
  ptr = ptr ->right;
  else
  ptr = ptr -> left;
  }
  if (x < parent -> data)
  parent -> left = temp;
  else
  parent -> right = temp;
  }
}
Void  bstree::in_order(node  *ptr)
 {
 if (ptr != NULL)
 {
```



```
in_order (ptr->left);
cout<<"  "<<ptr->data<<"  ";
in_order (ptr->right);
 }
}
void  bstree::pre_order(node *ptr)
{
if(ptr!=NULL)
 {
 cout<<"  "<<ptr->data<<"  ";
 pre_order (ptr->left);
 pre_order (ptr->right);
 }
}
void  bstree::post_order(node *ptr)
 {
 if(ptr!=NULL)
 {
 post_order (ptr->left);
 post_order (ptr->right);
 cout<<"  "<<ptr->data<<"  ";
 }
 }
 void  bstree::search(int key)
 {
 int  flag=0;
 node *temp;
 temp=root;
 if temp==NULL)
 {
 cout<<"\n BST  is  empty";
```

```
}
else
{
while(temp != NULL)
{
if (temp->data==key)
{
cout<<key<<"  "<<"is found"<<endl;
flag=1;
break;
}
if (temp->data >key)
temp=temp->left;
if (temp->data <key)
temp=temp->right;
}
}
if (flag==0)
cout<<key<<"  "<<"is not found"<<endl;
}
void main()
{
bstree bs;
int option,k;
clrscr();
cout<<"1. create()"<<endl;
cout<<"2. Insert()"<<endl;
cout<<"3. in_order()"<<endl;
cout<<"4. pre_order()"<<endl;
cout<<"5. post_order()"<<endl;
```

```
cout << "(6. search)" <<endl;
cout <<"7. exit " <<endl;
do
 {
 cout<<"\n enter your option: ";
 cin >> option;
 switch (option)
  {
  case 1:
  bs. create( ) ;
  break;
  case 2:
  bs. insert( );
   break;
  case 3:
  bs. in_order(root);
  break;
  case 4:
  bs. pre_order(root);
  break;
  case 5:
  bs. post_order(root);
   break;
  case 6:
  cout<<" enter the element to be searched:";
  cin>>k;
  bs. search(k);
  break;
  default :
  cout<<"\n enter a valid option: ";
   }
  }
 while (option!= 7);
 getch( );
}
```